MATH 102 - 17/18 FALL - WORKSHEET FOR WEEK 1

1. By recognizing a pattern, try to find a closed form (i.e. something like $a_n = \ldots$) for the following sequences. Obviously, many different answers are possible.
   a) $1,\ -2,\ 3,\ -4,\ 5,\ -6,\ \ldots$
   b) $40,\ 20,\ 5,\ \frac{5}{8},\ \frac{5}{128},\ \ldots$
   c) $1,\ 0,\ 1,\ 0,\ 1,\ 0,\ \ldots$
   d) $0,\ \frac{2}{3},\ \frac{4}{5},\ \frac{6}{7},\ \frac{8}{9},\ \frac{10}{11},\ \ldots$

2. Write the first 5 terms of the following sequences:
   a) $a_n = 2n$;
   b) $b_n = \cos(\pi n)$;
   c) $c_n = 3^{n-3}$;
   d) $d_n = b_n + c_n$.

3. For each of the following geometric series, identify the start term, the common ratio, compute the 2nd and 4th partial sum and, finally, decide if it is convergent or not. If yes, compute its sum.
   a) $\sum_{n=0}^{+\infty} \frac{2}{3^n}$;
   b) $\sum_{n=0}^{+\infty} \left(-\frac{2}{3}\right)^n$;
   c) $\sum_{n=0}^{+\infty} \left(\frac{4}{5}\right)^n$;
   d) $\sum_{n=0}^{+\infty} 2^n$;
   e) $\sum_{n=0}^{+\infty} 6\left(\frac{5}{6}\right)^{n+1}$;
   f) $\sum_{n=0}^{+\infty} \frac{(-2)^n}{3^{n-1}}$.

4. **Challenging (but not too much):** Your energetic instructor grades infinitely many exams. It takes 20 minutes to grade the first one. Then he speeds up and every other paper takes 9/10 of the time that is needed for the previous exam. For instance, the second paper is graded in $20 \times \frac{9}{10} = 18$ minutes, the third exam is graded in $18 \times \frac{9}{10} = 16.2$ minutes and so on. If the instructor starts at 14:00, when will he finish grading?

5. **[T/F]** Decide if the following statement are true or false. Explain (or give a counterexample for) each answer.
   a) The geometric series $\sum_{n=0}^{+\infty} \left(-\frac{3}{2}\right)^n$ converges to $\frac{1}{1+\frac{3}{2}} = \frac{2}{5}$.

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