Math 101 - 17/18 Spring - Worksheet for week 7

1. Find the tangent line approximation of $\frac{1}{3x}$ near $x = 1$.
2. Compute the linear approximation of $\sqrt{x+3}$ around 6. Use it to estimate $\sqrt{9.5}$.
3. Find and identify all critical points and inflection points for the functions below:
   (a) $h(x) = x^5 - 10x^3 - 8$;
   (b) $g(x) = x + \frac{1}{x}$;
   (c) $f(x) = (x^2 - 4)^7$.
4. Find the slope of the line tangent to the curve defined by $x^3 + 5x^2y + 2y^2 = 4y + 11$ at the point $(1, 2)$.
5. Find the tangent line to the curve defined by $\ln(xy) = 2x$ at the point $(1, e^2)$.
6. Find y'' if $x^2 + xy + y^2 = 1$.
7. **Challenging:** Try to find the slope of the tangent line to the curve defined by $x^2 + y^2 = 1$ at the points $(-1, -1)$ and $(1/\sqrt{2}, -1/\sqrt{2})$ in two different ways. Is something wrong somewhere, and why? And is there a third way to do it? And a fourth? How many of those methods can you use at $(1, 0)$?
8. **[T/F]** Decide if the following statements are true or false. Explain (or give a counterexample for) each answer.
   (a) If a point P is critical for a function f, then $f'(P) = 0$.
   (b) If the linear approximation of a function f at a point P is a horizontal line, then f has either a maximum or a minimum in P.
   (c) If a function f has continuous non-constant second derivative and two maxima in an interval, than it also has a minimum in that interval.

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